Charles Jones 2[nd] ed. *Introduction to Economic Growth*

Chapter 5: The Engine of growth

> As for the Arts of Delight and Ornament, they are best promoted by the greatest number of emulators. And it is more likely that one ingenious curious man may rather be found among 4 million than among 400 persons. . . .
>
> —WILLIAM PETTY, (cited in Simon (1981), p. 158).

The neoclassical growth model highlights technological progress as the engine of economic growth, and the previous chapter discussed in broad terms the economics of ideas and technology. In this chapter, we incorporate the insights from the previous chapters to develop an explicit theory of technological progress. The model we develop allows us to explore the engine of economic growth, thus addressing the second main question posed at the beginning of this book. We seek an understanding of why the advanced economies of the world, such as the United States, have grown at something like 2 percent per year for the last century. Where does the technological progress that underlies this growth come from? Why is the growth rate 2 percent per year instead of 1 percent or 10 percent? Can we expect this growth to continue, or is there some limit to economic growth?

Much of the work by economists to address these questions has been labeled *endogenous growth theory* or *new growth theory*. Instead of assuming that growth occurs because of automatic and unmodeled (exogenous) improvements in technology, the theory focuses on under-

standing the economic forces underlying technological progress. An important contribution of this work is the recognition that technological progress occurs as profit-maximizing firms or inventors seek out newer and better mousetraps. Adam Smith wrote that "it is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest" (Smith 1776 [1981], pp. 26–7). Similarly, it is the possibility of earning a profit that drives firms to develop a computer that can fit in your hand, a soft drink with only a single calorie, or a way to record TV programs and movies to be replayed at your convenience. In this way, improvements in technology, and the process of economic growth itself, are understood as an endogenous outcome of the economy.

The specific theory we will develop in this chapter was constructed by Paul Romer in a series of papers, including a 1990 paper titled "Endogeneous Technological Change."[1]

THE BASIC ELEMENTS OF THE MODEL

The Romer model endogenizes technological progress by introducing the search for new ideas by researchers interested in profiting from their inventions. The market structure and economic incentives that are at the heart of this process will be examined in detail in Section 5.2. First, though, we will outline the basic elements of the model and their implications for economic growth.

The model is designed to explain why and how the advanced countries of the world exhibit sustained growth. In contrast to the neoclassical models in earlier chapters, which could be applied to different countries, the model in this chapter describes the advanced countries of the world as a whole. Technological progress is driven by research and development (R&D) in the advanced world. In the next chapter we

[1] The version of the Romer model that we will present in this chapter is based on Jones (1995a). There is one key difference between the two models, which will be discussed at the appropriate time. Other notable contributions to the literature on R&D-based growth models include Grossman and Helpman (1991) and Aghion and Howitt (1992). These models are sometimes called Schumpeterian growth models, because they were anticipated by the work of Joseph Schumpeter in the late 1930s and early 1940s.

will explore the important process of technology transfer and why different economies have different levels of technology. For the moment, we will concern ourselves with how the world technological frontier is continually pushed outward.

As was the case with the Solow model, there are two main elements in the Romer model of endogenous technological change: an equation describing the production function and a set of equations describing how the inputs for the production function evolve over time. The main equations will be similar to the equations for the Solow model, with one important difference.

The aggregate production function in the Romer model describes how the capital stock, K, and labor, L_Y, combine to produce output, Y, using the stock of ideas, A:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}, \tag{5.1}$$

where α is a parameter between 0 and 1. For the moment, we take this production function as given; in Section 5.2, we will discuss in detail the market structure and the microfoundations of the economy that underlie this aggregate production function.

For a given level of technology, A, the production function in equation (5.1) exhibits constant returns to scale in K and L_Y. However, when we recognize that ideas (A) are also an input into production, then there are increasing returns. For example, once Steve Jobs and Steve Wozniak invented the plans for assembling personal computers, those plans (the "idea") did not need to be invented again. To double the production of personal computers, Jobs and Wozniak needed only to double the number of integrated circuits, semiconductors, etc., and find a larger garage. That is, the production function exhibits constant returns to scale with respect to the capital and labor inputs, and therefore must exhibit increasing returns with respect to all three inputs: if you double capital, labor, *and* the stock of ideas, then you will more than double output. As discussed in Chapter 4, the presence of increasing returns to scale results fundamentally from the nonrivalrous nature of ideas.

The accumulation equations for capital and labor are identical to those for the Solow model. Capital accumulates as people in the economy forego consumption at some given rate, s_K, and depreciates at the exogenous rate d:

$$\dot{K} = s_K Y - dK.$$

Labor, which is equivalent to the population, grows exponentially at some constant and exogenous rate n:

$$\frac{\dot{L}}{L} = n.$$

The key equation that is new relative to the neoclassical model is the equation describing technological progress. In the neoclassical model, the productivity term A grows exogenously at a constant rate. In the Romer model, growth in A is endogenized. How is this accomplished? The answer is with a production function for new ideas: just as more automobile workers can produce more cars, we assume that more researchers can produce more new ideas.

According to the Romer model, $A(t)$ is the stock of knowledge or the number of ideas that have been invented over the course of history up until time t. Then, $\dot{A}$ is the number of new ideas produced at any given point in time. In the simplest version of the model, $\dot{A}$ is equal to the number of people attempting to discover new ideas, L_A, multiplied by the rate at which they discover new ideas, $\bar{\delta}$:

$$\dot{A} = \bar{\delta} L_A. \tag{5.2}$$

The rate at which researchers discover new ideas might simply be a constant. On the other hand, one could imagine that it depends on the stock of ideas that have already been invented. For example, perhaps the invention of ideas in the past raises the productivity of researchers in the present. In this case, $\bar{\delta}$ would be an increasing function of A. The discovery of calculus, the invention of the laser, and the development of integrated circuits are examples of ideas that have increased the productivity of later research. On the other hand, perhaps the most obvious ideas are discovered first and subsequent ideas are increasingly difficult to discover. In this case, $\bar{\delta}$ would be a decreasing function of A.

This reasoning suggests modeling the rate at which new ideas are produced as

$$\bar{\delta} = \delta A^{\phi}, \tag{5.3}$$

where δ and ϕ are constants. In this equation, $\phi > 0$ indicates that the productivity of research increases with the stock of ideas that have already been discovered; $\phi < 0$ corresponds to the "fishing out" case in which the fish become harder to catch over time. Finally, $\phi = 0$ indicates that the tendency for the most obvious ideas to be discovered first exactly offsets the fact that old ideas may facilitate the discovery of new ideas—i.e., the productivity of research is independent of the stock of knowledge.

It is also possible that the average productivity of research depends on the number of people searching for new ideas at any point in time. For example, perhaps duplication of effort is more likely when there are more persons engaged in research. One way of modeling this possibility is to suppose that it is really $L_A{}^{\lambda}$, where λ is some parameter between 0 and 1, rather than L_A that enters the production function for new ideas. This, together with equations (5.3) and (5.2), suggests focusing on the following general production function for ideas:

$$\dot{A} = \delta L_A{}^{\lambda} A^{\phi}. \tag{5.4}$$

For reasons that will become clear, we will assume that $\phi < 1$.

Equations (5.2) and (5.4) illustrate a very important aspect of modeling economic growth.[2] Individual researchers, being small relative to the economy as a whole, take $\bar{\delta}$ as given and see constant returns to research. As in equation (5.2), an individual engaged in research creates $\bar{\delta}$ new ideas. In the economy as a whole, however, the production function for ideas may not be characterized by constant returns to scale. While $\bar{\delta}$ will change by only a minuscule amount in response to the actions of a single researcher, it clearly varies with aggregate research effort.[3] For example, $\lambda < 1$ may reflect an externality associated with duplication: some of the ideas created by an individual researcher may not be new to the economy as a whole. This is analogous to congestion on a highway. Each driver ignores the fact that his or her presence makes it slightly harder for other drivers to get where they are going. The effect of any single driver is negligible, but summed across all drivers, the effects can be important.

[2]This modeling technique will be explored again in Chapter 8 in the context of "AK" models of growth.

[3]Notice that the exact expression for $\bar{\delta}$, incorporating both duplication and knowledge spillovers, is $\bar{\delta} = \delta L_A{}^{\lambda-1} A^{\phi}$.

Similarly, the presence of A^{ϕ} is treated as external to the individual agent. Consider the case of $\phi > 0$, reflecting a positive knowledge spillover in research. The gains to society from the theory of gravitation far outweighed the benefit that Isaac Newton was able to capture. Much of the knowledge he created "spilled over" to future researchers. Of course, Newton himself also benefited from the knowledge created by previous scientists such as Kepler, as he recognized in the famous statement, "If I have seen farther than others, it is because I was standing on the shoulders of giants." With this in mind, we might refer to the externality associated with ϕ as the "standing on shoulders" effect, and by extension, the externality associated with λ as the "stepping on toes" effect.

Next, we need to discuss how resources are allocated in this economy. There are two key allocations. First, we assume (as before) that a constant fraction of output is invested in capital. Second, we have to decide how much labor works to produce output and how much works to produce ideas, recognizing that these two activities employ all of the labor in the economy:

$$L_Y + L_A = L.$$

In a more sophisticated model (and indeed, in Romer's original paper), the allocation of labor is determined by utility maximization and markets. However, it is again convenient to make the Solow-style assumption that the allocation of labor is constant; this assumption will be relaxed in Section 5.2. We assume that a constant fraction, $L_A/L = s_R$, of the labor force engages in R&D to produce new ideas, and the remaining fraction, $1 - s_R$, produces output.

Finally, the economy has some initial endowments when it begins. We assume the economy starts out with K_0 units of capital, L_0 units of labor, and A_0 ideas. This completes our setup of the model and we are ready to begin solving for some key endogenous variables, beginning with the long-run growth rate of this economy.

GROWTH IN THE ROMER MODEL

What is the growth rate in this model along a balanced growth path? Provided a constant fraction of the population is employed producing ideas (which we will show to be the case below), the model follows the neoclassical model in predicting that all per capita growth is due to technological progress. Letting lower-case letters denote per capita variables, and letting g_x denote the growth rate of some variable x along the balanced growth path, it is easy to show that

$$g_y = g_k = g_A.$$

That is, per capita output, the capital-labor ratio, and the stock of ideas must all grow at the same rate along a balanced growth path.[4] If there is no technological progress in the model, then there is no growth.

Therefore, the important question is "What is the rate of technological progress along a balanced growth path?" The answer to this question is found by rewriting the production function for ideas, equation (5.4). Dividing both sides of this equation by A yields

$$\frac{\dot{A}}{A} = \delta \frac{L_A{}^{\lambda}}{A^{1-\phi}}. \tag{5.5}$$

Along a balanced growth path, $\dot{A}/A \equiv g_A$ is constant. But this growth rate will be constant if and only if the numerator and the denominator of the right-hand side of equation (5.5) grow at the same rate. Taking logs and derivatives of both sides of this equation,

$$0 = \lambda \frac{\dot{L}_A}{L_A} - (1 - \phi)\frac{\dot{A}}{A}. \tag{5.6}$$

Along a balanced growth path, the growth rate of the number of researchers must be equal to the growth rate of the population—if it were higher, the number of researchers would eventually exceed the population, which is impossible. That is, $\dot{L}_A/L_A = n$. Substituting this into equation (5.6) yields

$$g_A = \frac{\lambda n}{1 - \phi}. \tag{5.7}$$

[4]To see this, follow the arguments we made in deriving equation (2.10) in Chapter 2. Intuitively, the capital-output ratio must be constant along a balanced growth path. Recognizing this fact, the production function implies that y and k must grow at the same rate as A.

Thus the long-run growth rate of this economy is determined by the parameters of the production function for ideas and the rate of growth of researchers, which is ultimately given by the population growth rate.

Several features of this equation deserve comment. First, what is the intuition for the equation? The intuition is most easily seen by considering the special case in which $\lambda = 1$ and $\phi = 0$ so that the productivity of researchers is the constant δ. In this case, there is no duplication problem in research and the productivity of a researcher today is independent of the stock of ideas that have been discovered in the past. The production function for ideas looks like

$$\dot{A} = \delta L_A.$$

Now suppose that the number of people engaged in the search for ideas is constant. Because δ is also constant, this economy generates a constant number of new ideas, δL_A, each period. To be more concrete, let's suppose $\delta L_A = 100$. The economy begins with some stock of ideas, A_0, generated by previous discoveries. Initially, the 100 new ideas per period may be a large fraction of the existing stock, A_0. Over time, though, the stock grows, and the 100 new ideas becomes a smaller and smaller fraction of the existing stock. Therefore, the *growth rate* of the stock of ideas falls over time, eventually approaching zero. Notice, however, that technological progress never ceases. The economy is always creating 100 new ideas. It is simply that these 100 new ideas shrink in comparison with the accumulated stock of ideas.

In order to generate exponential growth, the number of new ideas must be expanding over time. This occurs if the number of researchers is increasing—for example, because of world population growth. More researchers mean more ideas, sustaining growth in the model. In this case, the growth in ideas is clearly related to the growth in population, which explains the presence of population growth in equation (5.7). Phelps (1968) clarifies the intuition for this basic result with an enlightening example:

> One can hardly imagine, I think, how poor we would be today were it not for the rapid population growth of the past to which we owe the enormous number of technological advances enjoyed today.... If I could re-do the history of the world, halving population size each year from the beginning of time on some random basis, I would not do it for fear of losing Mozart in the process (pp. 511–512).

It is interesting to compare this result to the effect of population growth in the neoclassical growth model. There, for example, a higher population growth rate reduces the level of income along a balanced growth path. More people means that more capital is needed to keep K/L constant, but capital runs into diminishing returns. Here, an important additional effect exists. People are the key input to the creative process. A larger population generates more ideas, and because ideas are nonrivalrous, everyone in the economy benefits.

What evidence can be presented to support the contention that the per capita growth rate of the world economy depends on population growth? First, notice that this particular implication of the model is very difficult to test. We have already indicated that this model of the engine of growth is meant to describe the advanced countries of the world taken as a whole. Thus, we cannot use evidence on population growth *across* countries to test the model. In fact, we have already presented one of the most compelling pieces of evidence in Chapter 4. Recall the plot in Figure 4.4 of world population growth rates over the last 2,000 years. Sustained and rapid population growth is a rather recent phenomenon, just as is sustained and rapid growth in per capita output. Increases in the rate of population growth from the very low rate observed over most of history occurred at roughly the same time as the Industrial Revolution.

The result that the growth rate of the economy is tied to the growth rate of the population implies another seemingly strong result: if the population (or at least the number of researchers) stops growing, long-run growth ceases. What do we make of this prediction? Rephrasing the question slightly, if research effort in the world were constant over time, would economic growth eventually grind to a halt? This model suggests that it would. A constant research effort cannot continue the proportional increases in the stock of ideas needed to generate long-run growth.

Actually, there is one special case in which a constant research effort can sustain long-run growth, and this brings us to our second main comment about the model. The production function for ideas considered in the original Romer (1990) paper assumes that $\lambda = 1$ and $\phi = 1$. That is,

$$\dot{A} = \delta L_A A.$$

Rewriting the equation slightly, we can see that this version of the Romer model *will* generate sustained growth in the presence of a constant research effort:

$$\frac{\dot{A}}{A} = \delta L_A. \tag{5.8}$$

In this case, Romer assumes that the productivity of research is proportional to the existing stock of ideas: $\bar{\delta} = \delta A$. With this assumption, the productivity of researchers grows over time, even if the number of researchers is constant.

The advantage of this specification, however, is also its drawback. World research effort has increased enormously over the last forty years and even over the last century (see Figure 4.6 in Chapter 4 for a reminder of this fact). Since L_A is growing rapidly over time, the original Romer formulation in equation (5.8) predicts that the growth rate of the advanced economies should also have risen rapidly over the last forty years or the last century. We know this is far from the truth. The average growth rate of the U.S. economy, for example, has been very close to 1.8 percent per year for the last hundred years. This easily rejected prediction of the original Romer formulation is avoided by requiring that ϕ is less than one, which returns us to the results associated with equation (5.7).[5]

Notice that nothing in this reasoning rules out increasing returns in research or positive knowledge spillovers. The knowledge spillover parameter, ϕ, may be positive and quite large. What the reasoning points out is that the somewhat arbitrary case of $\phi = 1$ is strongly rejected by empirical observation.[6]

Our last comment about the growth implications of this model of technology is that the results are similar to the neoclassical model in one important way. In the neoclassical model, changes in government policy and changes in the investment rate have no long-run effect on economic growth. This result was not surprising once we recognized that all growth in the neoclassical model was due to exogenous technological progress. In this model with endogenous technological progress, however, we have the same result. The long-run growth rate is invari-

[5] This point is made in Jones (1995a).

[6] The same evidence also rules out values of $\phi > 1$. Such values would generate accelerating growth rates even with a constant population!

ant to changes in the investment rate, and even to changes in the share of the population that is employed in research. This is seen by noting that none of the parameters in equation (5.7) is affected when, say, the investment rate or the R&D share of labor is changed. Instead, these policies affect the growth rate along a transition path to the new steady state altering the *level* of income. That is, even after we endogenize technology in this model, the long-run growth rate cannot be manipulated by policy makers using conventional policies such as subsidies to R&D.

GROWTH EFFECTS VERSUS LEVEL EFFECTS

The fact that standard policies cannot affect long-run growth is *not* a feature of the original Romer model, nor of many other idea-based growth models that followed, including Grossman and Helpman (1991) and Aghion and Howitt (1992). Much of the theoretical work in new growth theory has sought to develop models in which policy changes *can* have effects on long-run growth.

The idea-based models in which changes in policy can permanently increase the growth rate of the economy all rely on the assumption that $\phi = 1$, or its equivalent. As shown above, this assumption generates the counterfactual prediction that growth rates should accelerate over time with a growing population. Jones (1995a) generalized these models to the case of $\phi < 1$ to eliminate this defect and showed the somewhat surprising implication that this eliminates the long-run growth effects of policy as well. We will discuss these issues in more detail in Chapter 8.

COMPARATIVE STATICS: A PERMANENT INCREASE IN THE R&D SHARE

What happens to the advanced economies of the world if the share of the population searching for new ideas increases permanently? For example, suppose there is a government subsidy for R&D that increases the fraction of the labor force doing research.

An important feature of the model we have just developed is that many policy changes (or comparative statics) can be analyzed with techniques we have already developed. Why? Notice that technological progress in the model can be analyzed by itself — it doesn't depend

on capital or output, but only on the labor force and the share of the population devoted to research. Once the growth rate of A is constant, the model behaves just like the Solow model with exogenous technological progress. Therefore, our analysis proceeds in two steps. First, we consider what happens to technological progress and to the stock of ideas after the increase in R&D intensity occurs. Second, we analyze the model as we did the Solow model, in steps familiar from Chapter 2. Before we proceed, it is worth noting that the analysis of changes that do not affect technology, such as an increase in the investment rate, is exactly like the analysis of the Solow model.

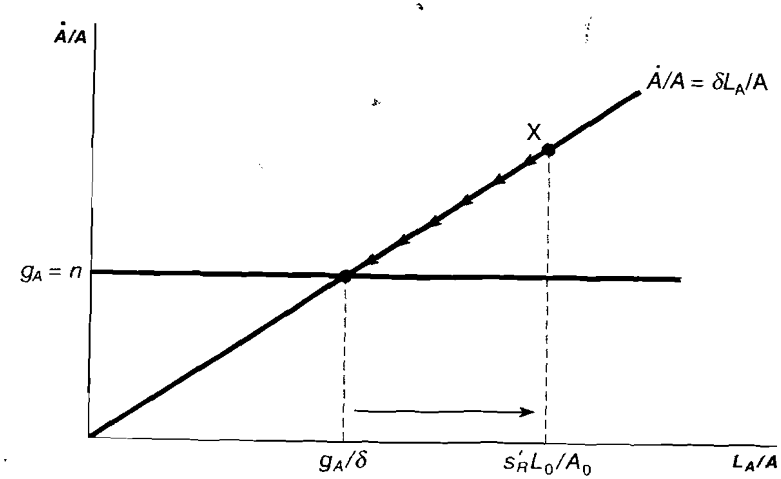
Now consider what happens if the share of the population engaged in research increases permanently. To simplify things slightly, let's assume that $\lambda = 1$ and $\phi = 0$ again; none of the results are qualitatively affected by this assumption. It is helpful to rewrite equation (5.5) as

$$\frac{\dot{A}}{A} = \delta \frac{s_R L}{A}, \tag{5.9}$$

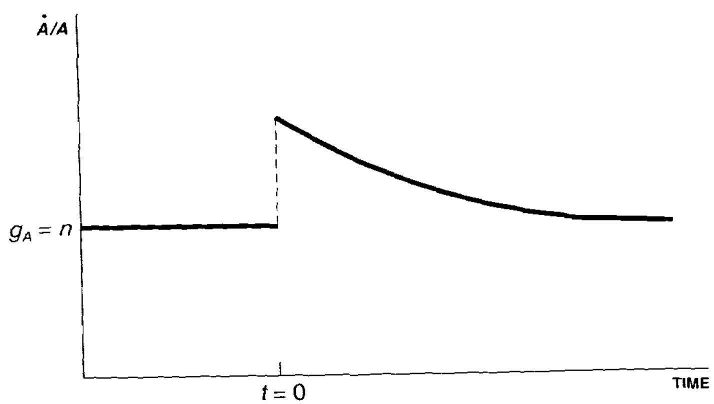
where s_R is the share of the population engaged in R&D, so that $L_A = s_R L$.

Figure 5.1 shows what happens to technological progress when s_R increases permanently to s_R', assuming the economy begins in steady

TECHNOLOGICAL PROGRESS: AN INCREASE IN THE R&D SHARE



$\dot{A}/A$ OVER TIME



state. In steady state, the economy grows along a balanced growth path at the rate of technological progress, g_A, which happens to equal the rate of population growth under our simplifying assumptions. The ratic L_A/A is therefore equal to g_A/δ. Suppose the increase in s_R occurs a time $t = 0$. With a population of L_0, the number of researchers increase as s_R increases, so that the ratio L_A/A jumps to a higher level. Th additional researchers produce an increased number of new ideas, sc the growth rate of technology is also higher at this point. This situatio corresponds to the point labeled "X" in the figure. At X, technologica progress $\dot{A}/A$ exceeds population growth n, so the ratio L_A/A decline over time, as indicated by the arrows. As this ratio declines, the rat of technological change gradually falls also, until the economy return to the balanced growth path where $g_A = n$. Therefore, a permanen increase in the share of the population devoted to research raises th rate of technological progress temporarily, but not in the long run. Thi behavior is depicted in Figure 5.2.

What happens to the level of technology in this economy? Figure 5. answers this question. The level of technology is growing along a ba anced growth path at rate g_A until time $t = 0$. At this time, the growt rate increases and the level of technology rises faster than before. Ove

time, however, the growth rate falls until it returns to g_A. The *level* of technology is permanently higher as a result of the permanent increase in R&D. Notice that a permanent increase in s_R in the Romer model generates transition dynamics that are qualitatively similar to the dynamics generated by an increase in the investment rate in the Solow model.

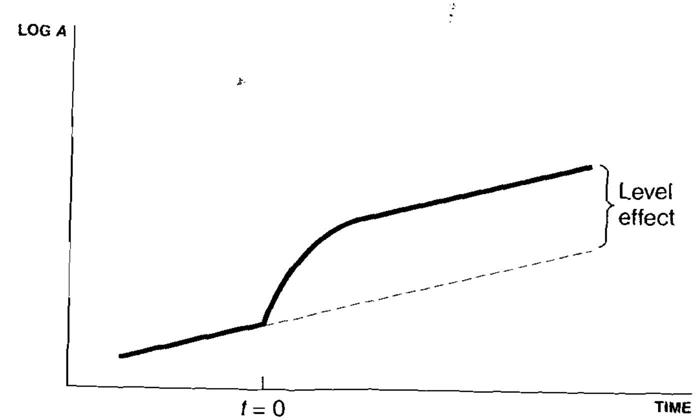
Now that we know what happens to technology over time, we can analyze the remainder of the model in a Solow framework. The long-run growth rate of the model is constant, so much of the algebra that we used in analyzing the Solow model applies. For example, the ratio y/A is constant along a balanced growth path and is given by an equation similar to equation (2.13):

$$\left(\frac{y}{A}\right)^* = \left(\frac{s_K}{n + g_A + d}\right)^{\alpha/(1-\alpha)} (1 - s_R). \tag{5.10}$$

The only difference is the presence of the term $1 - s_R$, which adjusts for the difference between output per worker, L_Y, and output per capita, L.

Notice that along a balanced growth path, equation (5.9) can be solved for the level of A in terms of the labor force:

THE LEVEL OF TECHNOLOGY OVER TIME



Notice that along a balanced growth path, equation (5.9) can be solved for the level of A in terms of the labor force:

$$A = \frac{\delta s_R L}{g_A}.$$

Combining this equation with (5.10), we get

$$y^*(t) = \left(\frac{s_K}{n + g_A + d}\right)^{\alpha/(1-\alpha)} (1 - s_R)\frac{\delta s_R}{g_A} L(t). \tag{5.11}$$

In this simple version of the model, per capita output is proportional to the population of the (world) economy along a balanced growth path. In other words, the model exhibits a *scale effect* in levels: a larger world economy will be a richer world economy. This scale effect arises fundamentally from the nonrivalrous nature of ideas: a larger economy provides a larger market for an idea, raising the return to research (a demand effect). In addition, a more populous world economy simply has more potential creators of ideas in the first place (a supply effect).

The other terms in equation (5.11) are readily interpreted. The first term is familiar from the original Solow model. Economies that invest more in capital will be richer, for example. Two terms involve the share of labor devoted to research, s_R. The first time s_R appears, it enters negatively to reflect the fact that more researchers mean fewer workers producing output. The second time, it enters positively to reflect the fact that more researchers mean more ideas, which increases the productivity of the economy.

THE ECONOMICS OF THE MODEL

The first half of this chapter has analyzed the Romer model without discussing the economics underlying the model. A number of economists in the 1960s developed models with similar macroeconomic features.[7] However, the development of the microfoundations of such models had to wait until the 1980s when economists better understood how to model imperfect competition in a general equilibrium setting.[8] In

[7] For example, Uzawa (1965), Phelps (1966), Shell (1967), and Nordhaus (1969).

[8] Key steps in this understanding were accomplished by Spence (1976), Dixit and Stiglitz (1977), and Ethier (1982).

fact, one of the important contributions of Romer (1990) was to explain exactly how to construct an economy of profit-maximizing agents that endogenizes technological progress. The intuition behind this insight was developed in Chapter 4. Developing the mathematics is the subject of the remainder of this section. Because this section is somewhat difficult, some readers may wish to skip to Section 5.3.

The Romer economy consists of three sectors: a final-goods sector, an intermediate-goods sector, and a research sector. The reason for two of the sectors should be clear: some firms must produce output and some firms must produce ideas. The reason for the intermediate-goods sector is related to the presence of increasing returns discussed in Chapter 4. Each of these sectors will be discussed in turn. Briefly, the research sector creates new ideas, which take the form of new varieties of capital goods — new computer chips, fax machines, or printing presses. The research sector sells the exclusive right to produce a specific capital good to an intermediate-goods firm. The intermediate-goods firm, as a monopolist, manufactures the capital good and sells it to the final-goods sector, which produces output.

THE FINAL-GOODS SECTOR

The final-goods sector of the Romer economy is very much like the final-goods sector of the Solow model. It consists of a large number of perfectly competitive firms that combine labor and capital to produce a homogeneous output good, Y. The production function is specified in a slightly different way, though, to reflect the fact that there is more than one capital good in the model:

$$Y = L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha}.$$

Output, Y, is produced using labor, L_Y, and a number of different capital goods, x_j, which we will also call "intermediate goods." At any point in time, A measures the number of capital goods that are available to be used in the final-goods sector, and firms in the final-goods sector take this number as given. Inventions or ideas in the model correspond to the creation of new capital goods that can be used by the final-goods sector to produce output.

Notice that this production function can be rewritten as

$$Y = L_Y^{1-\alpha}x_1^\alpha + L_Y^{1-\alpha}x_2^\alpha + \cdots + L_Y^{1-\alpha}x_A^\alpha,$$

and it is easy to see that, for a given A, the production function exhibits constant returns to scale; doubling the amount of labor and the amount of each capital good will exactly double output.

It turns out for technical reasons to be easier to analyze the model if we replace the summation in the production function with an integral:

$$Y = L_Y^{1-\alpha}\int_0^A x_j^\alpha\, dj.$$

Then, A measures the range of capital goods that are available to the final-goods sector, and this range is the interval on the real line $[0, A]$. The basic interpretation of this equation, though, is unaffected by this technicality.

With constant returns to scale, the number of firms cannot be pinned down, so we will assume there are a large number of identical firms producing final output and that perfect competition prevails in this sector. We will also normalize the price of the final output, Y, to unity.

Firms in the final-goods sector have to decide how much labor and how much of each capital good to use in producing output. They do this by solving the profit-maximization problem:

$$\max_{L_Y, x_j} L_Y^{1-\alpha}\int_0^A x_j^\alpha\, dj - wL_Y - \int_0^A p_j x_j\, dj,$$

where p_j is the rental price for capital good j and w is the wage paid for labor. The first-order conditions characterizing the solution to this problem are

$$w = (1-\alpha)\frac{Y}{L_Y} \tag{5.12}$$

and

$$p_j = \alpha L_Y^{1-\alpha}x_j^{\alpha-1}, \tag{5.13}$$

where this second condition applies to each capital good j. The first condition says that firms hire labor until the marginal product of labor equals the wage. The second condition says the same thing, but for capital goods: firms rent capital goods until the marginal product of each kind of capital equals the rental price, p_j. To see the intuition for these equations, suppose the marginal product of a capital good were higher than its rental price. Then the firm should rent another unit; the output produced will more than pay for the rental price. If the marginal product is below the rental price, then the firm can increase profits by reducing the amount of capital used.

THE INTERMEDIATE-GOODS SECTOR

The intermediate-goods sector consists of monopolists who produce the capital goods that are sold to the final-goods sector. These firms gain their monopoly power by purchasing the design for a specific capital good from the research sector. Because of patent protection, only one firm manufactures each capital good.

Once the design for a particular capital good has been purchased (a fixed cost), the intermediate-goods firm produces the capital good with a very simple production function: one unit of raw capital can be automatically translated into one unit of the capital good. The profit maximization problem for an intermediate goods firm is then

$$\max_{x_j} \pi_j = p_j(x_j)x_j - rx_j,$$

where $p_j(x)$ is the demand function for the capital good given in equation (5.13). The first-order condition for this problem, dropping the j subscripts, is

$$p'(x)x + p(x) - r = 0.$$

Rewriting this equation we get

$$p'(x)\frac{x}{p} + 1 = \frac{r}{p},$$

which implies that

$$p = \frac{1}{1 + \frac{p'(x)x}{p}}r.$$

Finally, the elasticity, $p'(x)x/p$, can be calculated from the demand curve in equation (5.13). It is equal to $\alpha - 1$, so the intermediate-goods firm charges a price that is simply a markup over marginal cost, r:

$$p = \frac{1}{\alpha} r.$$

This is the solution for each monopolist, so that all capital goods sell for the same price. Because the demand functions in equation (5.13) are also the same, each capital good is employed by the final-goods firms in the same amount: $x_j = x$. Therefore, each capital-goods firm earns the same profit. With some algebra, one can show that this profit is given by

$$\pi = \alpha(1-\alpha)\frac{Y}{A}. \tag{5.14}$$

Finally, the total demand for capital from the intermediate-goods firms must equal the total capital stock in the economy:

$$\int_0^A x_j dj = K.$$

Since the capital goods are each used in the same amount, x, this equation can be used to determine x:

$$x = \frac{K}{A}. \tag{5.15}$$

The final-goods production function can be rewritten, using the fact that $x_j = x$, as

$$Y = AL_Y{}^{1-\alpha}x^{\alpha},$$

and substituting from equation (5.15) reveals that

$$\begin{aligned} Y &= AL_Y{}^{1-\alpha}A^{-\alpha}K^{\alpha} \\ &= K^{\alpha}(AL_Y)^{1-\alpha}. \end{aligned} \tag{5.16}$$

That is, we see that the production technology for the final-goods sector generates the same aggregate production function used throughout this book. In particular, this is the aggregate production function used in equation (5.1).

THE RESEARCH SECTOR

Much of the analysis of the research sector has already been provided. The research sector is essentially like gold mining in the wild West in the mid-nineteenth century. Anyone is free to "prospect" for ideas, and the reward for prospecting is the discovery of a "nugget" that can be sold. Ideas in this model are designs for new capital goods: a faster computer chip, a method for genetically altering corn to make it more resistant to pests, or a new way to organize movie theaters. These designs can be thought of as instructions that explain how to transform a unit of raw capital into a unit of a new capital good. New designs are discovered according to equation (5.4).

When a new design is discovered, the inventor receives a patent from the government for the exclusive right to produce the new capital good. (To simplify the analysis, we assume that the patent lasts forever.) The inventor sells the patent to an intermediate-goods firm and uses the proceeds to consume and save, just like any other agent in the model. But what is the price of a patent for a new design?

We assume that anyone can bid for the patent. How much will a potential bidder be willing to pay? The answer is the present discounted value of the profits to be earned by an intermediate-goods firm. Any less, and someone would be willing to bid higher; any more, and no one would be willing to bid. Let P_A be the price of a new design, this present discounted value. How does P_A change over time? The answer lies in an extremely useful line of reasoning in economics and finance called the method of *arbitrage*.

The arbitrage argument goes as follows. Suppose I have some money to invest for one period. I have two options. First, I can put the money in the "bank" (in this model, this is equivalent to purchasing a unit of capital) and earn the interest rate r. Alternatively, I can purchase a patent for one period, earn the profits that period, and then sell the patent. In equilibrium, it must be the case that the rate of return from both of these investments is the same. If not, everyone would jump at the more profitable investment, driving its return down. Mathematically, the *arbitrage equation* states that the returns are the same:

$$rP_A = \pi + \dot{P}_A. \tag{5.17}$$

The left-hand side of this equation is the interest earned from investing

P_A in the bank; the right-hand side is the profits plus the capital gain or loss that results from the change in the price of the patent. These two must be equal in equilibrium.

Rewriting equation (5.17) slightly,

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A}.$$

Along a balanced growth path, r is constant.[9] Therefore, π/P_A must be constant also, which means that π and P_A have to grow at the same rate; this rate turns out to be the population growth rate, n.[10] Therefore, the arbitrage equation implies that

$$P_A = \frac{\pi}{r-n}. \tag{5.18}$$

This equation gives the price of a patent along a balanced growth path.

SOLVING THE MODEL

We have now described the market structure and the microeconomics underlying the basic equations given in Section 5.1. The model is somewhat complicated, but several features that were discussed in Chapter 4 are worth noting. First, the aggregate production function exhibits increasing returns. There are constant returns to K and L, but increasing returns once we note that ideas, A, are also an input to production. Second, the increasing returns require imperfect competition. This appears in the model in the intermediate-goods sector. Firms in this sector are monopolists, and capital goods sell at a price that is greater than marginal cost. However, the profits earned by these firms are extracted by the inventors, and these profits simply compensate the inventors for the time they spend "prospecting" for new designs. This framework is called *monopolistic competition*. There are no economic profits in the model; all rents compensate some factor input. Finally, once we depart from the world of perfect competition there is no reason to think that

[9] The interest rate r is constant for the usual reasons. It will be the price at which the supply of capital is equal to the demand for capital, and will be proportional to Y/K.

[10] To see this, recall from equation (5.14) that π is proportional to Y/A. Per capita output, y, and A grow at the same rate, so that Y/A will grow at the rate of population growth.

markets yield the "best of all possible worlds." This last point is one that we develop more carefully in the next section.

We have already solved for the growth rate of the economy in steady state. The part of the model that remains to be solved is the allocation of labor between research and the final-goods sector. Rather than assuming s_R is constant, we let it be determined endogenously by the model.

Once again, the concept of arbitrage enters. It must be the case that, at the margin, individuals in this simplified model are indifferent between working in the final-goods sector and working in the research sector. Labor working in the final-goods sector earns a wage that is equal to its marginal product in that sector, as given in equation (5.12):

$$w_Y = (1-\alpha)\frac{Y}{L_Y}.$$

Researchers earn a wage based on the value of the designs they discover. We will assume that researchers take their productivity in the research sector, $\bar{\delta}$, as given. They do not recognize that productivity falls as more labor enters because of duplication, and they do not internalize the knowledge spillover associated with ϕ. Therefore, the wage earned by labor in the research sector is equal to its marginal product, $\bar{\delta}$, multiplied by the value of the new ideas created, P_A:

$$w_R = \bar{\delta} P_A.$$

Because there is free entry into both the research sector and the final goods sector, these wages must be the same: $w_Y = w_R$. This condition, with some algebra shown in the appendix to this chapter, reveals that the share of the population that works in the research sector, s_R, is given by

$$s_R = \frac{1}{1 + \frac{r-n}{\alpha g_A}}. \tag{5.19}$$

Notice that the faster the economy grows (the higher is g_A), the higher the fraction of the population that works in research. The higher the discount rate that applies to current profits to get the present discounted value ($r-n$), the lower the fraction working in research.[11]

[11] One can eliminate the interest rate from this equation by noting that $r = \alpha^2 Y/K$ and getting the capital-output ratio from the capital accumulation equation: $Y/K = (n+g+d)/s_K$.

With some algebra, one can show that the interest rate in this economy is given by $r = \alpha^2 Y/K$. Notice that this is *less* than the marginal product of capital, which from equation (5.16) is the familiar $\alpha Y/K$. This difference reflects an important point. In the Solow model with perfect competition and constant returns to scale, all factors are paid their marginal products: $r = \alpha Y/K$, $w = (1-\alpha)Y/L$, and therefore $rK + wL = Y$. In the Romer model, however, production in the economy is characterized by increasing returns and all factors cannot be paid their marginal products. This is clear from the Solow example just given: because $rK + wL = Y$, there is no output in the Solow economy remaining to compensate individuals for their effort in creating new A. *This* is what necessitates imperfect competition in the model. Here, capital is paid less than its marginal product, and the remainder is used to compensate researchers for the creation of new ideas.

OPTIMAL R&D

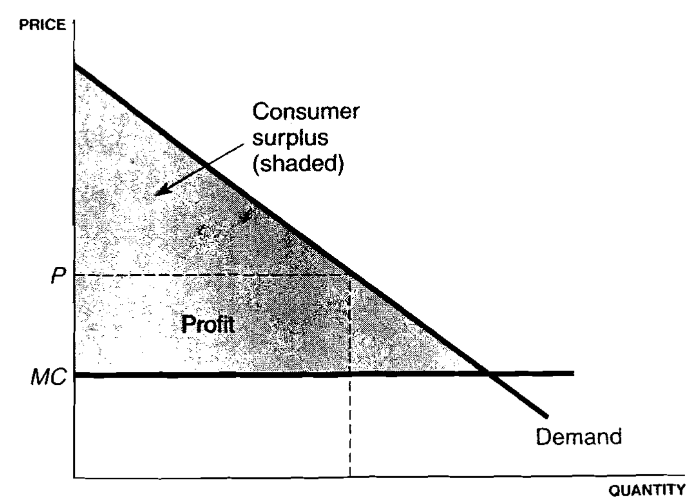
Is the share of the population that works in research optimal? In general, the answer to this question in the Romer model is no. In this case, the markets do not induce the right amount of labor to work in research. Why not? Where does Adam Smith's invisible hand go wrong?

There are three distortions to research in the model that cause s_R to differ from its optimal level. Two of the distortions are easy to see from the production function for ideas. First, the market values research according to the stream of profits that are earned from the new design. What the market misses, though, is that the new invention may affect the productivity of future research. Recall that $\phi > 0$ implies that the productivity of research increases with the stock of ideas. The problem here is one of a missing market: researchers are not compensated for their contribution toward improving the productivity of future researchers. For example, subsequent generations did not reward Isaac Newton sufficiently for inventing calculus. Therefore, with $\phi > 0$, there is a tendency, other things being equal, for the market to provide too little research. This distortion is often called a "knowledge spillover" because some of the knowledge created "spills over" to other researchers. This is the "standing on shoulders" effect referred to earlier. In this sense, it is very much like a classic positive externality: if the bees that a farmer raises for honey provide an extra benefit to the community that the farmer doesn't capture (they pollinate the apple trees in the surrounding area), the market will underprovide honey bees.[12]

The second distortion, the "stepping on toes" effect, is also a classic externality. It occurs because researchers do not take into account the fact that they lower research productivity through duplication when λ is less than 1. In this case, however, the externality is negative. Therefore, the market tends to provide too much research, other things being equal.

Finally, the third distortion can be called a "consumer-surplus effect." The intuition for this distortion is simple and can be seen by considering a standard monopoly problem, as in Figure 5.4. An inventor of a new design captures the monopoly profit shown in the figure. However, the potential gain to society from inventing the good is the entire consumer-surplus triangle above the marginal cost of production (MC). The incentive to innovate, the monopoly profit, is less than the gain to society, and this effect tends to generate too little innovation, other things being equal.

FIGURE 5.4 THE "CONSUMER-SURPLUS EFFECT"



[12]On the other hand, if $\phi < 0$, then the reverse could be true.

In practice, these distortions can be very large. Consider the consumer surplus associated with basic inventions such as the cure for malaria or cholera or the discovery of calculus. For these inventions, associated with "basic science," the knowledge spillovers and the consumer-surplus effects are generally felt to be so large that the government funds basic research in universities and research centers.

These distortions may also be important even for R&D undertaken by firms. Consider the consumer surplus benefits from the invention of the telephone, electric power, the laser, and the transistor. A substantial literature in economics, led by Zvi Griliches, Edwin Mansfield, and others, seeks to estimate the "social" rate of return to research performed by firms. Griliches (1991) reviews this literature and finds social rates of return on the order of 40 to 60 percent, far exceeding private rates of return. As an empirical matter, this suggests that the positive externalities of research outweigh the negative externalities so that the market, even in the presence of the modern patent system, tends to provide too little research.

A final comment on imperfect competition and monopolies is in order. Classical economic theory argues that monopolies are bad for welfare and efficiency because they create "deadweight losses" in the economy. This reasoning underlies regulations designed to prevent firms from pricing above marginal cost. In contrast, the economics of ideas suggests that it is critical that firms be allowed to price above marginal cost. It is exactly this wedge that provides the profits that are the incentive for firms to innovate. In deciding antitrust issues, modern regulation of imperfect competition has to weigh the deadweight losses against the incentive to innovate.

5.4 SUMMARY

Technological progress is the engine of economic growth. In this chapter, we have endogenized the process by which technological change occurs. Instead of "manna from heaven," technological progress arises as individuals seek out new ideas in an effort to capture some of the social gain these new ideas generate in the form of profit. Better mousetraps get invented and marketed because people will pay a premium for a better way to catch mice.

In Chapter 4, we showed that the nonrivalrous nature of ideas implies that production is characterized by increasing returns to scale. In this chapter, this implication served to illustrate the general importance of scale in the economy. Specifically, the growth rate of world technology is tied to the growth rate of the population. A larger number of researchers can create a larger number of ideas, and it is this general principle that generates per capita growth.

As in the Solow model, comparative statics in this model (such as an increase in the investment rate or an increase in the share of the labor force engaged in R&D) generate *level effects* rather than long-run growth effects. For example, a government subsidy that increases the share of labor in research will typically increase the growth rate of the economy, but only temporarily, as the economy transits to a higher level of income.

The results of this chapter match up nicely with the historical evidence documented in Chapter 4. Consider broadly the history of economic growth in reverse chronological order. The Romer model is clearly meant to describe the evolution of technology since the establishment of intellectual property rights. It is the presence of patents and copyrights that enables inventors to earn profits to cover the initial costs of developing new ideas. In the last century (or two), the world economy has witnessed sustained, rapid growth in population, technology, and per capita income never before seen in history.

Consider how the model economy would behave in the absence of property rights. In this case, innovators would be unable to earn the profits that encourage them to undertake research in the first place, so that no research would take place. With no research, no new ideas would be created, technology would be constant, and there would be no per capita growth in the economy. Broadly speaking, just such a situation prevailed in the world prior to the Industrial Revolution.[13]

Finally, a large body of research suggests that social returns to innovation remain well above private returns. Although the "prizes" that the market offers to potential innovators are substantial, these prizes

[13] There were, of course, very notable scientific and technological advances before 1760, but these were intermittent and there was little sustained growth. What did occur might be attributed to individual curiosity, government rewards, or public funding (such as the prize for the chronometer and the support for astronomical observatories).

still fall far short of the total gain to society from innovations. This gap between social and private returns suggests that large gains are still available from the creation of new mechanisms designed to encourage research. Mechanisms like the patent system are themselves ideas, and there is no reason to think the best ideas have already been discovered.

APPENDIX: SOLVING FOR THE R&D SHARE

The share of the population that works in research, s_R, is obtained by setting the wage in the final-goods sector equal to the wage in research:

$$\bar{\delta} P_A = (1-\alpha)\frac{Y}{L_Y}.$$

Substituting for P_A from equation (5.18),

$$\bar{\delta}\frac{\pi}{r-n} = (1-\alpha)\frac{Y}{L_Y}.$$

Recall that π is proportional to Y/A in equation (5.14):

$$\frac{\bar{\delta}}{r-n}\alpha(1-\alpha)\frac{Y}{A} = (1-\alpha)\frac{Y}{L_Y}.$$

Several terms cancel, leaving

$$\frac{\alpha}{r-n}\frac{\bar{\delta}}{A} = \frac{1}{L_Y}.$$

Finally, notice that $\dot{A}/A = \bar{\delta}L_A/A$, so that $\bar{\delta}/A = g_A/L_A$ along a balanced growth path. With this substitution,

$$\frac{\alpha g_A}{r-n} = \frac{L_A}{L_Y}.$$

Notice that L_A/L_Y is just $s_R/(1-s_R)$. Solving the equation for s_R then reveals

$$s_R = \frac{1}{1+\frac{r-n}{\alpha g_A}},$$

as reported in equation (5.19).

EXERCISES

1. *An increase in the productivity of research.* Suppose there is a one-time increase in the productivity of research, represented by an increase in δ in Figure 5.1. What happens to the growth rate and the level of technology over time?

2. *Too much of a good thing?* Consider the level of per capita income along a balanced growth path given by equation (5.11). Find the value for s_R that maximizes output per worker along a balanced growth path for this example. Why is it possible to do too much R&D according to this criterion?

3. *The future of economic growth* (from Jones (2002)). Recall from Figure 4.6 and the discussion surrounding this figure in Chapter 4 that the number of scientists and engineers engaged in R&D has been growing faster than the rate of population growth in the advanced economies of the world. To take some plausible numbers, assume population growth is 1 percent and the growth rate of researchers is 3 percent per year. Assume that $\dot{A}/A$ has been constant at about 2 percent per year.

   (a) Using equation (5.6), calculate an estimate of $\lambda/(1-\phi)$.

   (b) Using this estimate and equation (5.7), calculate an estimate of the long-run steady-state growth rate of the world economy.

   (c) Why does your estimate of long-run steady-state growth differ from the 2 percent rate of growth of A observed historically?

   (d) Does the fact that many developing countries are starting to engage in R&D change this calculation?

4. *The share of the surplus appropriated by inventors* (from Kremer 1998). In Figure 5.4, find the ratio of the profit captured by the monopolist to the total potential consumer surplus available if the good were priced at marginal cost. Assume that marginal cost is constant at c and the demand curve is linear: $Q = a - bP$, where a, b, and c are positive constants with $a - bc > 0$.